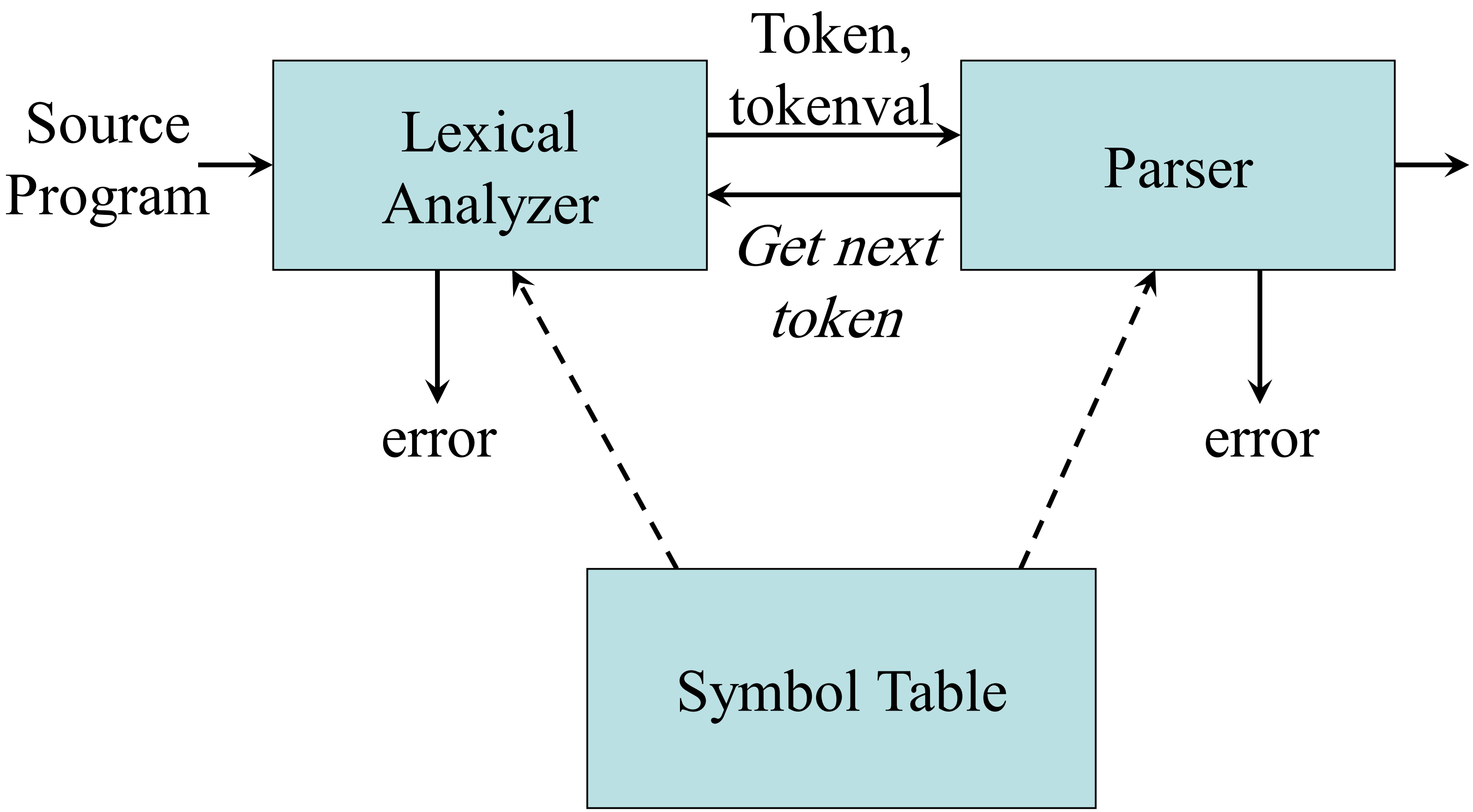
Lexical Analysis and Lexical Analyzer Generators

Chapter 3

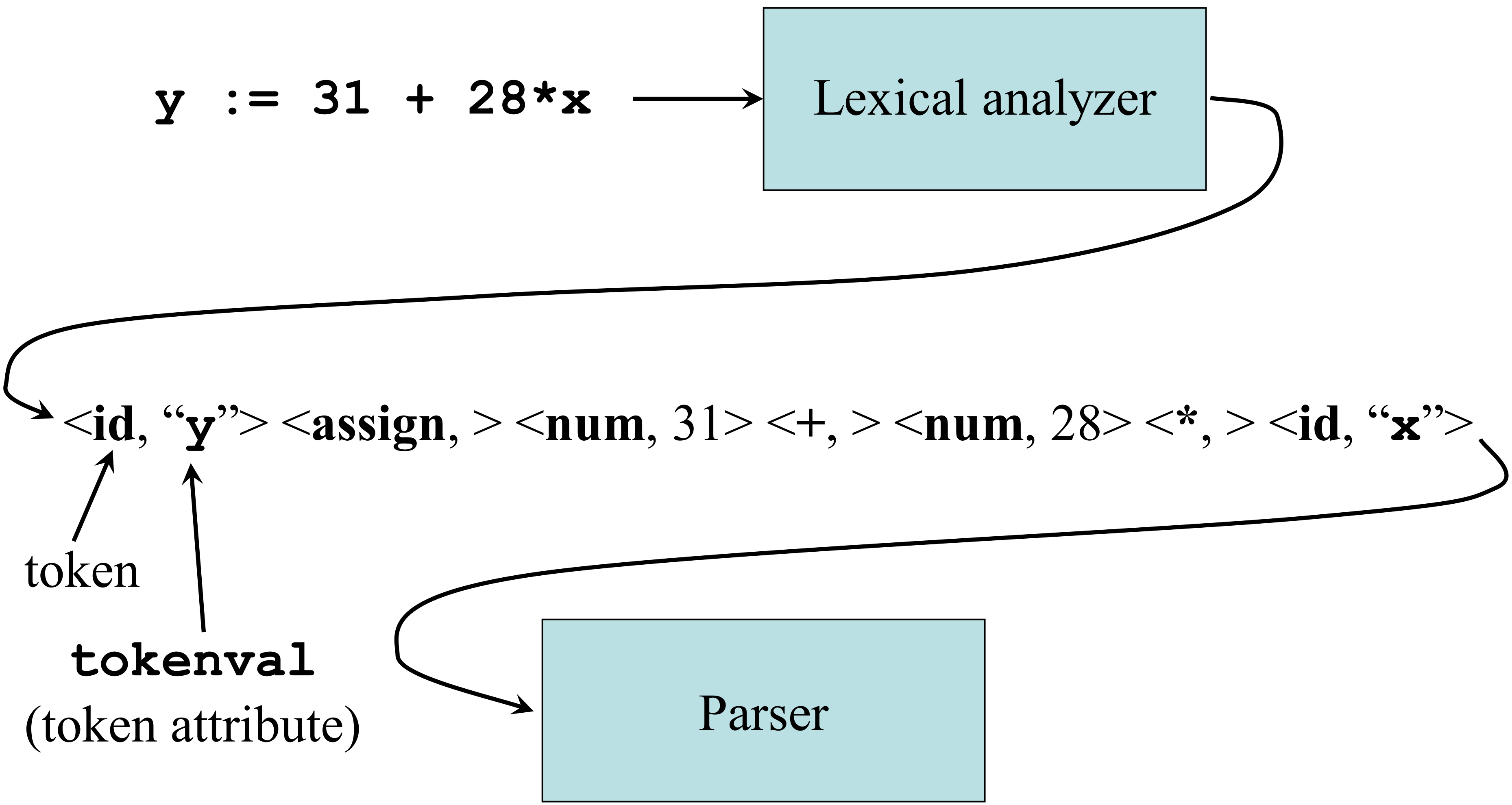
The Reason Why Lexical Analysis is a Separate Phase

- Simplifies the design of the compiler
  - LL(1) or LR(1) with 1 lookahead would not be possible
- Provides efficient implementation
  - Systematic techniques to implement lexical analyzers by hand or automatically
  - Stream buffering methods to scan input
- Improves portability
  - Non-standard symbols and alternate character encodings can be more easily translated

Interaction of the Lexical Analyzer with the Parser



Attributes of Tokens



Tokens, Patterns, and Lexemes

- A *token* is a classification of lexical units
  - For example: **id** and **num**
- *Lexemes* are the specific character strings that make up a token
  - For example: **`abc`** and **`123`**
- *Patterns* are rules describing the set of lexemes belonging to a token
  - For example: "*letter followed by letters and digits*" and "*non-empty sequence of digits*"

Specification of Patterns for Tokens: Terminology

- An *alphabet* Σ is a finite set of symbols (characters)
- A *string s* is a finite sequence of symbols from Σ
  - $|s|$ denotes the length of string *s*
  - ε denotes the empty string, thus $|\varepsilon| = 0$
- A *language* is a specific set of strings over some fixed alphabet Σ

Specification of Patterns for Tokens: String Operations

- The *concatenation* of two strings x and y is denoted by xy
- The *exponentation* of a string s is defined by

  $s^0 = \varepsilon$

  $s^i = s^{i-1} s$ for $i > 0$

  (note that $s\varepsilon = \varepsilon s = s$)

Specification of Patterns for Tokens: Language Operations

- *Union*

  $L \cup M = \{s \mid s \in L \text{ or } s \in M\}$

- *Concatenation*

  $LM = \{xy \mid x \in L \text{ and } y \in M\}$

- *Exponentiation*

  $L^0 = \{\varepsilon\}$; $L^i = L^{i-1}L$

- *Kleene closure*

  $L^* = \cup_{i=0,\dots,\infty} L^i$

- *Positive closure*

  $L^+ = \cup_{i=1,\dots,\infty} L^i$

Specification of Patterns for Tokens: Regular Expressions

- Basis symbols:
  - ε is a regular expression denoting language $\{\varepsilon\}$
  - $a \in \Sigma$ is a regular expression denoting $\{a\}$
- If r and s are regular expressions denoting languages $L(r)$ and $M(s)$ respectively, then
  - $r \mid s$ is a regular expression denoting $L(r) \cup M(s)$
  - rs is a regular expression denoting $L(r)M(s)$
  - r^* is a regular expression denoting $L(r)^*$
  - (r) is a regular expression denoting $L(r)$
- A language defined by a regular expression is called a *regular set*

Specification of Patterns for Tokens: Regular Definitions

- Naming convention for regular expressions:

  $d_1 \to r_1$
  $d_2 \to r_2$
  …
  $d_n \to r_n$

  where r_i is a regular expression over

  $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

- Each d_j in r_i is textually substituted in r_i

Specification of Patterns for Tokens: Regular Definitions

- Example:

letter → **`A`** | **`B`** | … | **`Z`** | **`a`** | **`b`** | … | **`z`**
digit → **`0`** | **`1`** | … | **`9`**
id → **letter** (**letter** | **digit**)*

- Cannot use recursion, this is illegal:

digits → **digit digits** | **digit**

Specification of Patterns for Tokens: Notational Shorthands

- We frequently use the following shorthands:

$$r^+ = rr^*$$

$$r? = r \mid \varepsilon$$

$[\texttt{a-z}] = \texttt{a} \mid \texttt{b} \mid \texttt{c} \mid \ldots \mid \texttt{z}$

- For example:

digit $\rightarrow$ [`0-9`]

num $\rightarrow$ **digit**$^+$ (. **digit**$^+$)? (**E** (+|-)? **digit**$^+$)?

Regular Definitions and Grammars

Grammar

stmt → **if** *expr* **then** *stmt*
  | **if** *expr* **then** *stmt* **else** *stmt*
  | ε

expr → *term* **relop** *term*
  | *term*

term → **id**
  | **num**

Regular definitions

if → `if`

then → `then`

else → `else`

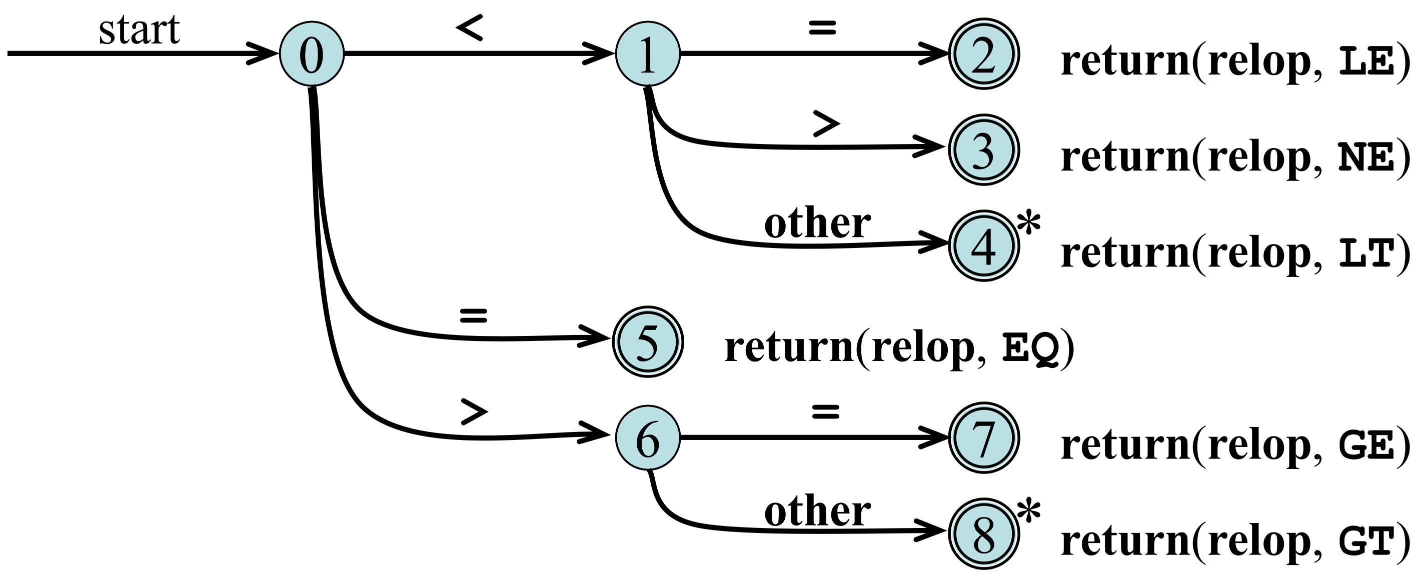
relop → `<` | `<=` | `<>` | `>` | `>=` | `=`

id → **letter (letter** | **digit)***

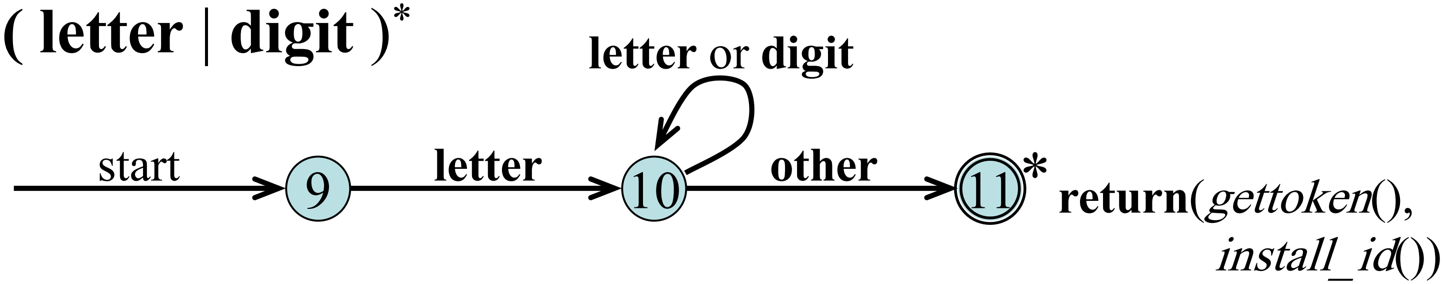
num → **digit**$^+$ (. **digit**$^+$)? (**E** (+|-)? **digit**$^+$)?

Implementing a Scanner Using Transition Diagrams

relop $\rightarrow$ **< | <= | <> | > | >= | =**



id $\rightarrow$ **letter (letter | digit)***



Implementing a Scanner Using Transition Diagrams (Code)

```
token nexttoken()
{ while (1) {
    switch (state) {
    case 0: c = nextchar();
      if (c==blank || c==tab || c==newline) {
        state = 0;
        lexeme_beginning++;
      }
      else if (c=='<') state = 1;
      else if (c=='=') state = 5;
      else if (c=='>') state = 6;
      else state = fail();
      break;
     case 1:
      …
     case 9: c = nextchar();
      if (isletter(c)) state = 10;
      else state = fail();
      break;
     case 10: c = nextchar();
      if (isletter(c)) state = 10;
      else if (isdigit(c)) state = 10;
      else state = 11;
      break;
     …
```

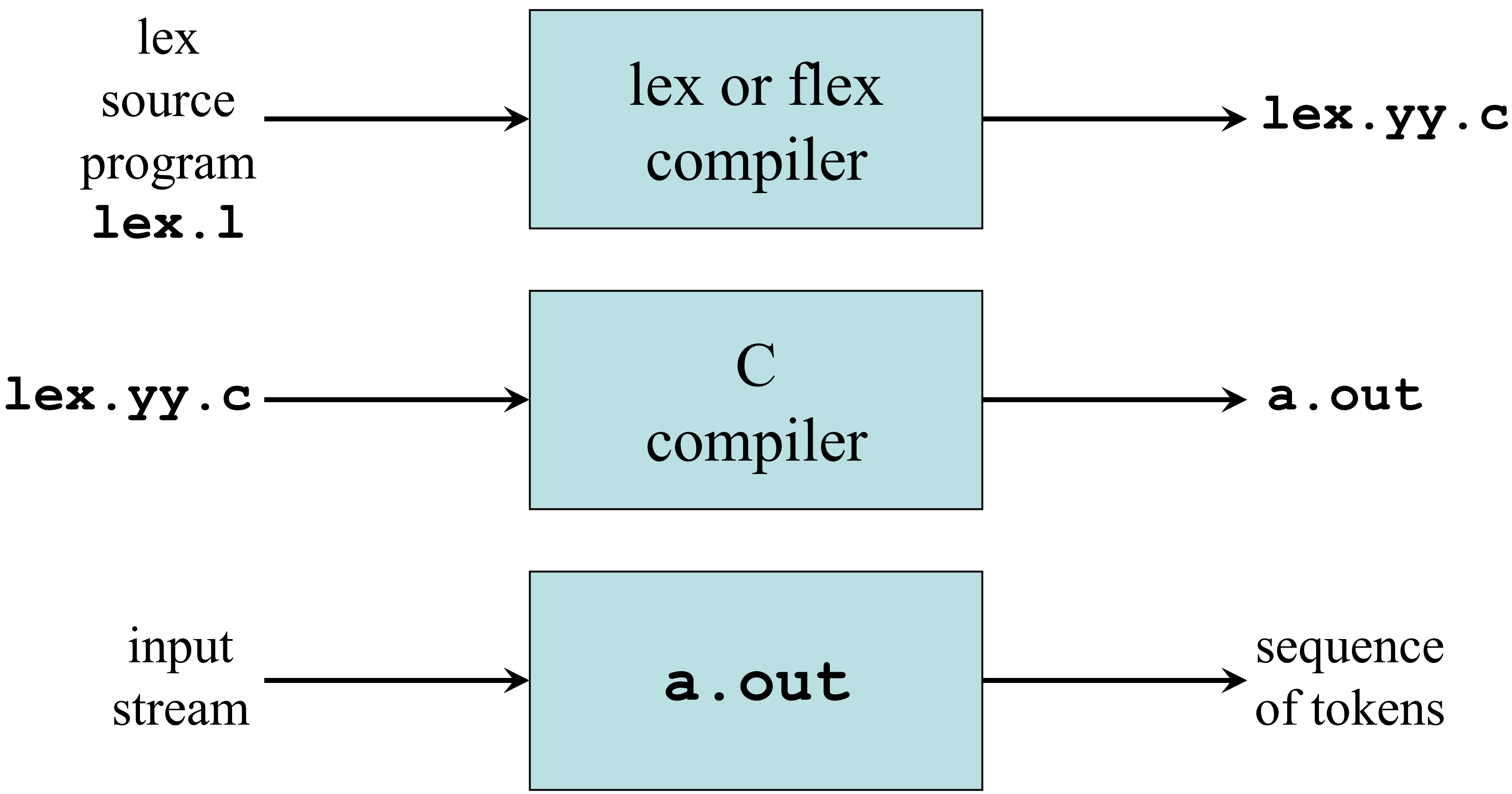
Decides what other start state is applicable

```
int fail()
{ forward = token_beginning;
  swith (start) {
  case  0: start =  9; break;
  case  9: start = 12; break;
  case 12: start = 20; break;
  case 20: start = 25; break;
  case 25: recover(); break;
  default: /* error */
  }
  return start;
}
```

The Lex and Flex Scanner Generators

- *Lex* and its newer cousin *flex* are scanner generators
- Systematically translate regular definitions into C source code for efficient scanning
- Generated code is easy to integrate in C applications

Creating a Lexical Analyzer with Lex and Flex



Lex Specification

- A *lex specification* consists of three parts:

  regular definitions, C declarations in **%{ %}**
  %%
  translation rules
  %%
  user-defined auxiliary procedures

- The *translation rules* are of the form:

  p_1 { $action_1$ }
  p_2 { $action_2$ }
  …
  p_n { $action_n$ }

Regular Expressions in Lex

`x`	match the character **`x`**
`\.`	match the character **`.`**
"*string*"	match contents of string of characters
`.`	match any character except newline
`^`	match beginning of a line
`$`	match the end of a line
`[xyz]`	match one character **`x`**, **`y`**, or **`z`** (use **`\`** to escape **`-`**)
`[^xyz]`	match any character except **`x`**, **`y`**, and **`z`**
`[a-z]`	match one of **`a`** to **`z`**
r**`*`**	closure (match zero or more occurrences)
r**`+`**	positive closure (match one or more occurrences)
r**`?`**	optional (match zero or one occurrence)
$r_1 r_2$	match r_1 then r_2 (concatenation)
$r_1 \mid r_2$	match r_1 or r_2 (union)
`(` r **`)`**	grouping
$r_1 \backslash r_2$	match r_1 when followed by r_2
`{`d**`}`**	match the regular expression defined by d

Example Lex Specification 1

Contains the matching lexeme

Translation rules

```
%{
#include <stdio.h>
%}
%%
[0-9]+  { printf(“%s\n”, yytext); }
.|\n    { }
%%
main()
{ yylex();
}
```

Invokes the lexical analyzer

```
lex spec.l
gcc lex.yy.c -ll
./a.out < spec.l
```

Example Lex Specification 2

Translation rules

Regular definition

```
%{
#include <stdio.h>
int ch = 0, wd = 0, nl = 0;
%}
delim       [ \t]+
%%
\n          { ch++; wd++; nl++; }
^{delim}    { ch+=yyleng; }
{delim}     { ch+=yyleng; wd++; }
.           { ch++; }
%%
main()
{ yylex();
  printf("%8d%8d%8d\n", nl, wd, ch);
}
```

Example Lex Specification 3

Translation rules

Regular definitions

```
%{
#include <stdio.h>
%}
digit     [0-9]
letter    [A-Za-z]
id        {letter}({letter}|{digit})*
%%
{digit}+  { printf("number: %s\n", yytext); }
{id}      { printf("ident: %s\n", yytext); }
.         { printf("other: %s\n", yytext); }
%%
main()
{ yylex();
}
```

Example Lex Specification 4

```
%{ /* definitions of manifest constants */
#define LT (256)
…
%}
delim       [ \t\n]
ws          {delim}+
letter      [A-Za-z]
digit       [0-9]
id          {letter}({letter}|{digit})*
number      {digit}+(\.{digit}+)?(E[+\-]?{digit}+)?
%%
{ws}        { }
if          {return IF;}
then        {return THEN;}
else        {return ELSE;}
{id}        {yylval = install_id(); return ID;}
{number}    {yylval = install_num(); return NUMBER;}
"<"         {yylval = LT; return RELOP;}
"<="        {yylval = LE; return RELOP;}
"="         {yylval = EQ; return RELOP;}
"<>"        {yylval = NE; return RELOP;}
">"         {yylval = GT; return RELOP;}
">="        {yylval = GE; return RELOP;}
%%
int install_id()
…
```

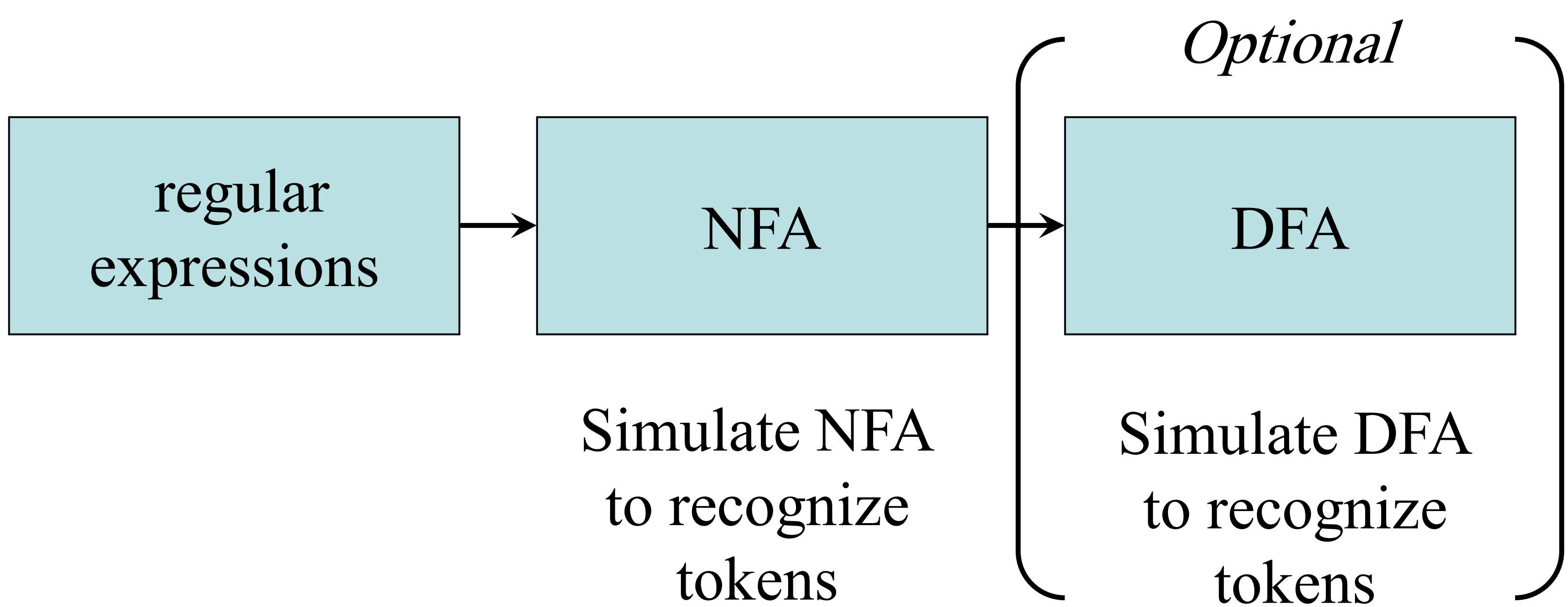
Return token to parser

Token attribute

Install **`yytext`** as identifier in symbol table

Design of a Lexical Analyzer Generator

- Translate regular expressions to NFA
- Translate NFA to an efficient DFA



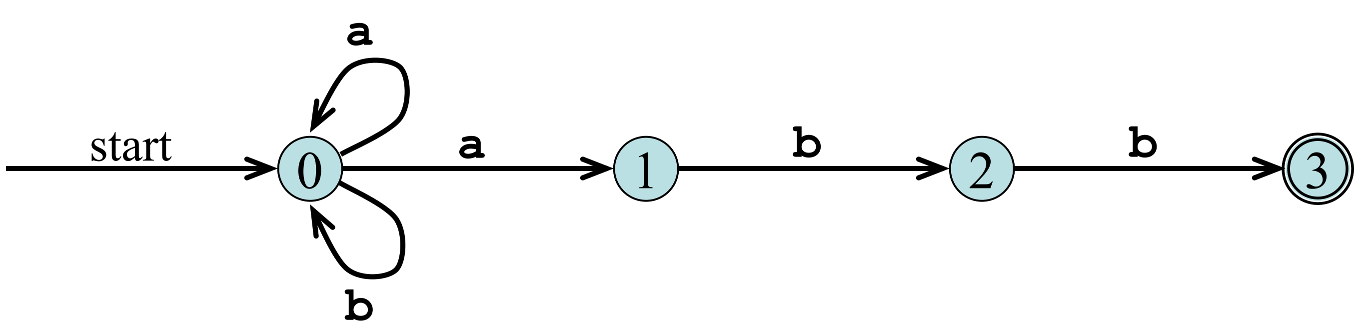
Nondeterministic Finite Automata

- Definition: an NFA is a 5-tuple (S,Σ,δ,s_0,F) where

  S is a finite set of *states*
  Σ is a finite set of *input symbol alphabet*
  δ is a *mapping* from $S\times\Sigma$ to a set of states
  $s_0 \in S$ is the *start state*
  $F \subseteq S$ is the set of *accepting* (or *final*) *states*

Transition Graph

- An NFA can be diagrammatically represented by a labeled directed graph called a *transition graph*



$S = \{0,1,2,3\}$
$\Sigma = \{\mathbf{a},\mathbf{b}\}$
$s_0 = 0$
$F = \{3\}$

Transition Table

- The mapping δ of an NFA can be represented in a *transition table*

$$\delta(0,\mathbf{a}) = \{0,1\}$$
$$\delta(0,\mathbf{b}) = \{0\}$$
$$\delta(1,\mathbf{b}) = \{2\}$$
$$\delta(2,\mathbf{b}) = \{3\}$$

$\longrightarrow$

State	*Input* **a**	*Input* **b**
0	{0, 1}	{0}
1		{2}
2		{3}

The Language Defined by an NFA

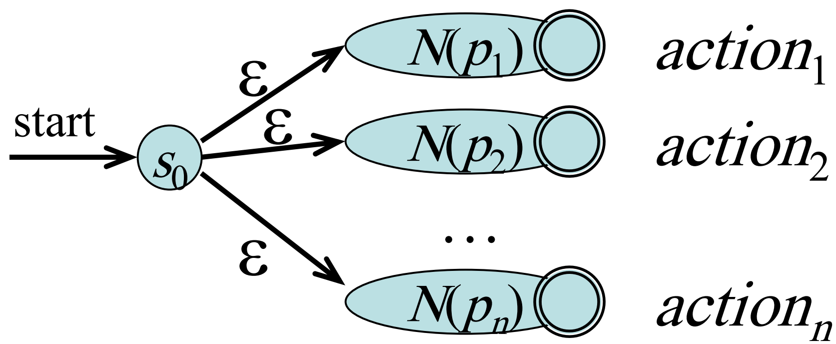
- An NFA *accepts* an input string *x* **iff** there is some path with edges labeled with symbols from *x* in sequence from the start state to some accepting state in the transition graph
- A state transition from one state to another on the path is called a *move*
- The *language defined by* an NFA is the set of input strings it accepts, such as (**`a`**|**`b`**)***`abb`** for the example NFA

Design of a Lexical Analyzer Generator: RE to NFA to DFA

Lex specification with regular expressions

p_1 { $action_1$ }
p_2 { $action_2$ }
…
p_n { $action_n$ }

NFA

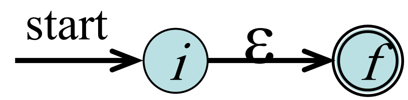


Subset construction (optional)

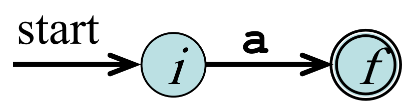
DFA

From Regular Expression to NFA (Thompson's Construction)

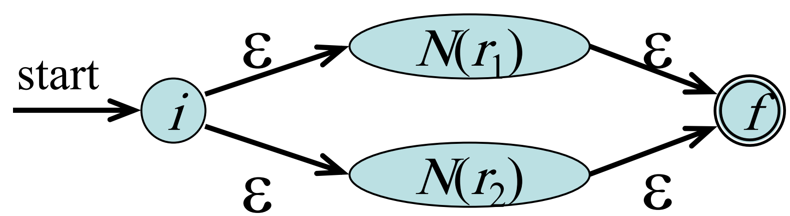
ε



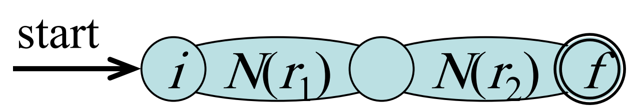
a



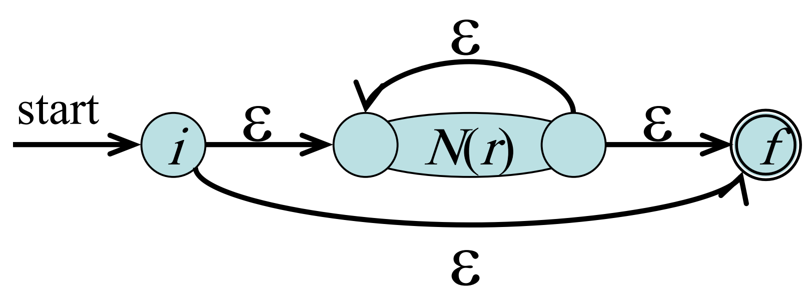
$r_1 \mid r_2$



$r_1 r_2$

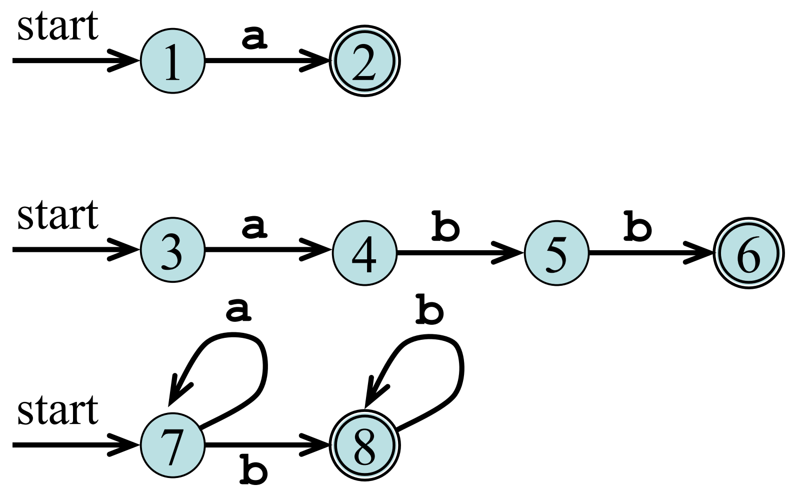


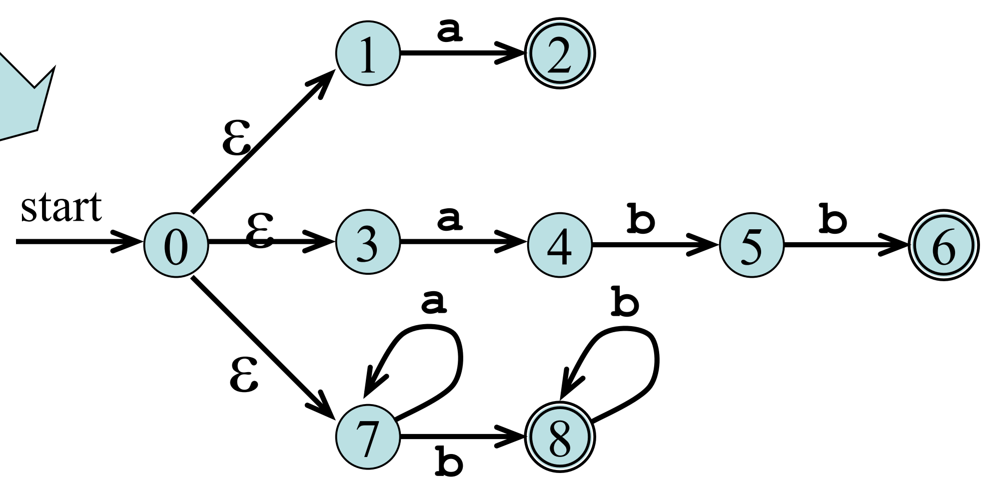
r^*



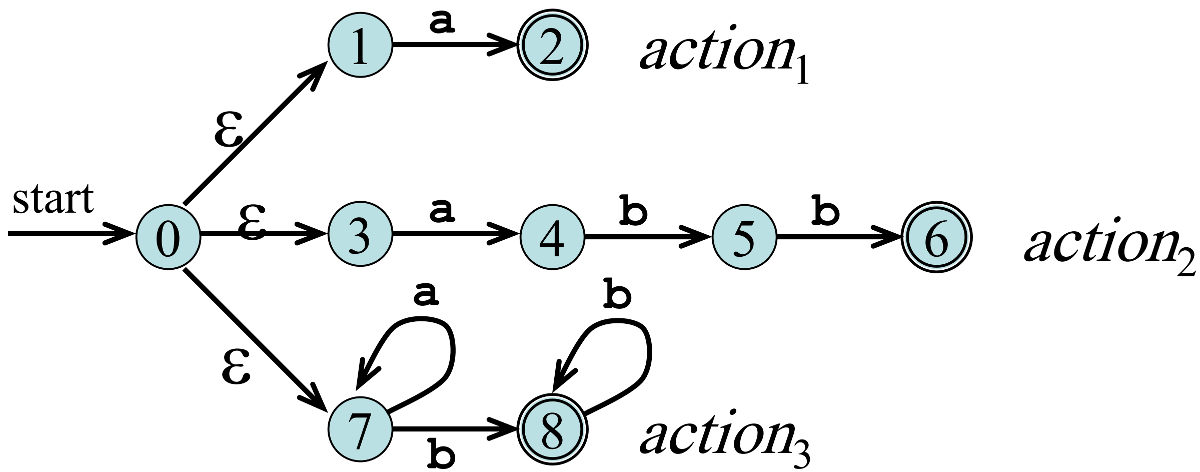
Combining the NFAs of a Set of Regular Expressions

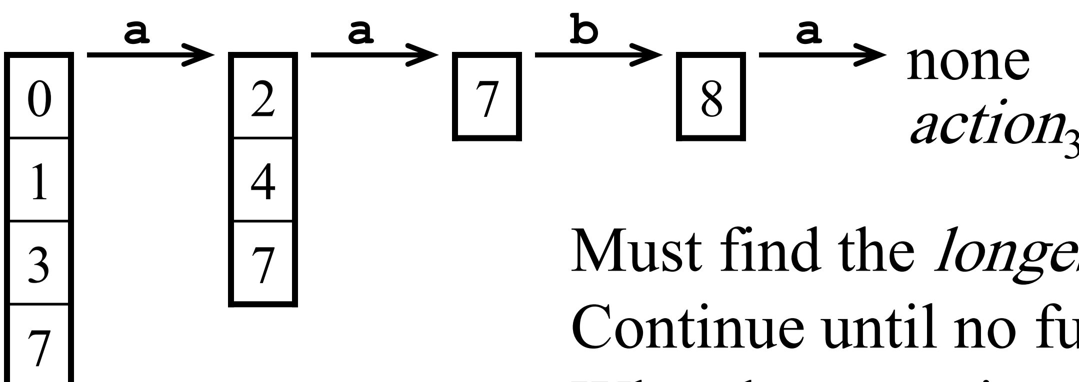
a	{ $action_1$ }
abb	{ $action_2$ }
a*b+	{ $action_3$ }





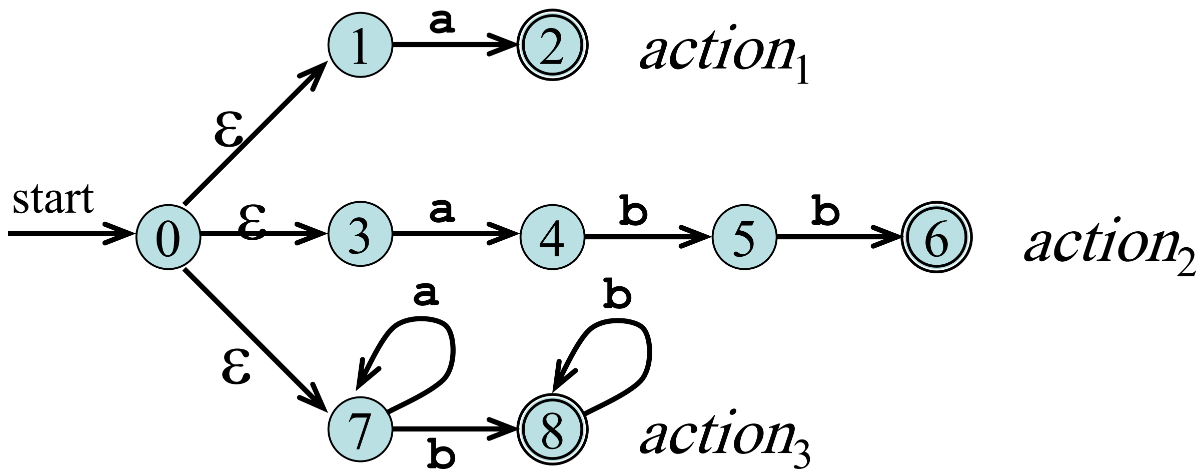
Simulating the Combined NFA Example 1

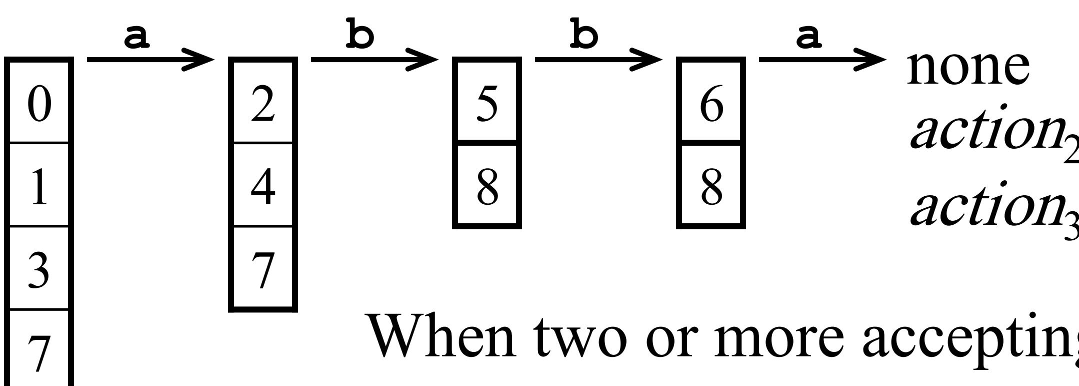




Must find the *longest match*:
Continue until no further moves are possible
When last state is accepting: execute action

Simulating the Combined NFA Example 2





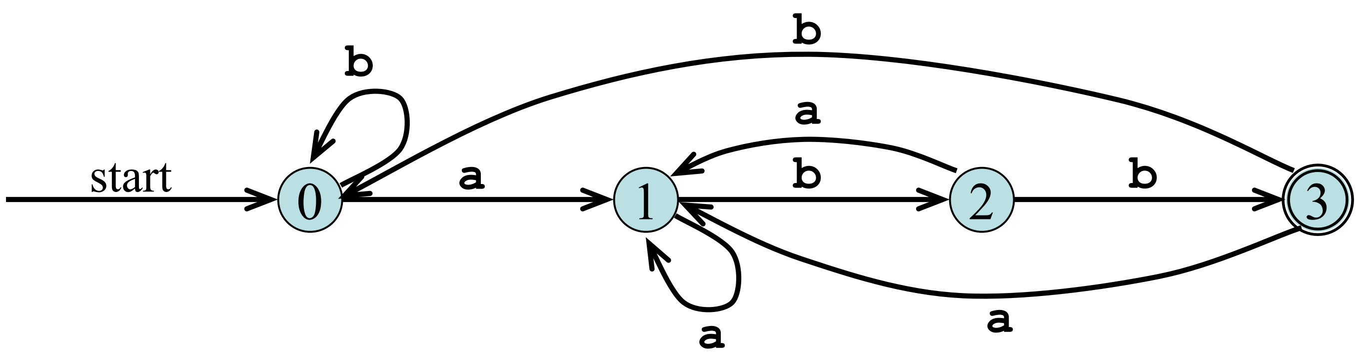
When two or more accepting states are reached, the first action given in the Lex specification is executed

Deterministic Finite Automata

- A *deterministic finite automaton* is a special case of an NFA
  - No state has an ε-transition
  - For each state *s* and input symbol *a* there is at most one edge labeled *a* leaving *s*
- Each entry in the transition table is a single state
  - At most one path exists to accept a string
  - Simulation algorithm is simple

Example DFA

A DFA that accepts (**a**|**b**)***abb**



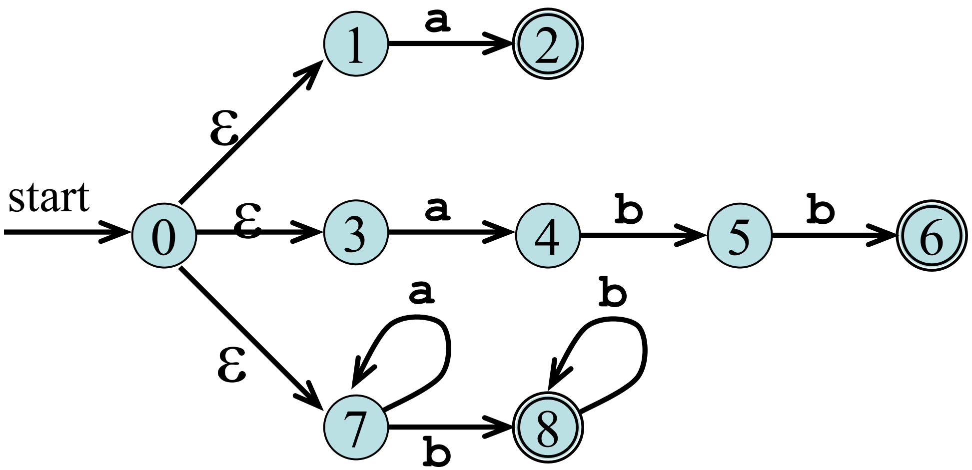
Conversion of an NFA into a DFA

- The *subset construction algorithm* converts an NFA into a DFA using:

  $\varepsilon\text{-}closure(s) = \{s\} \cup \{t \mid s \rightarrow_\varepsilon \ldots \rightarrow_\varepsilon t\}$
  $\varepsilon\text{-}closure(T) = \cup_{s \in T} \varepsilon\text{-}closure(s)$
  $move(T,a) = \{t \mid s \rightarrow_a t \text{ and } s \in T\}$

- The algorithm produces:
  Dstates is the set of states of the new DFA consisting of sets of states of the NFA
  Dtran is the transition table of the new DFA

ε-*closure* and *move* Examples



ε-*closure*$(\{0\}) = \{0,1,3,7\}$

move$(\{0,1,3,7\},\mathbf{a}) = \{2,4,7\}$

ε-*closure*$(\{2,4,7\}) = \{2,4,7\}$

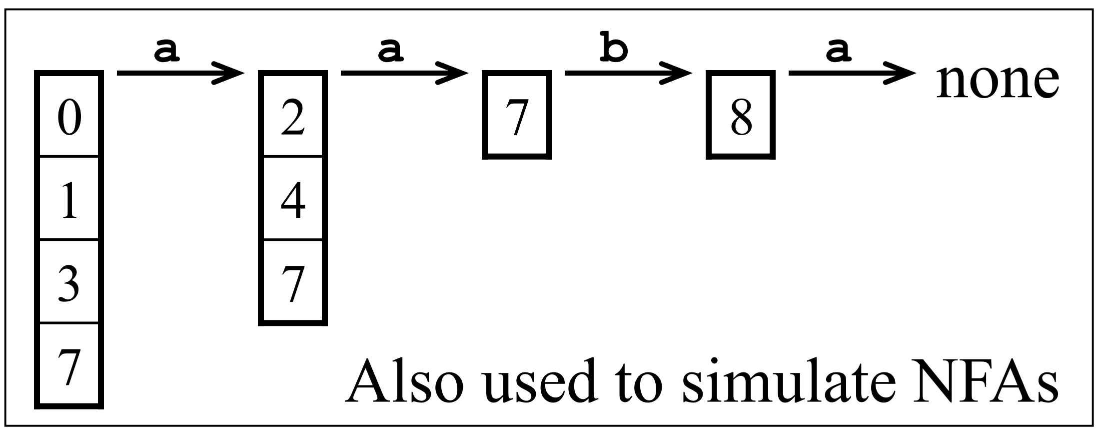
move$(\{2,4,7\},\mathbf{a}) = \{7\}$

ε-*closure*$(\{7\}) = \{7\}$

move$(\{7\},\mathbf{b}) = \{8\}$

ε-*closure*$(\{8\}) = \{8\}$

move$(\{8\},\mathbf{a}) = \varnothing$



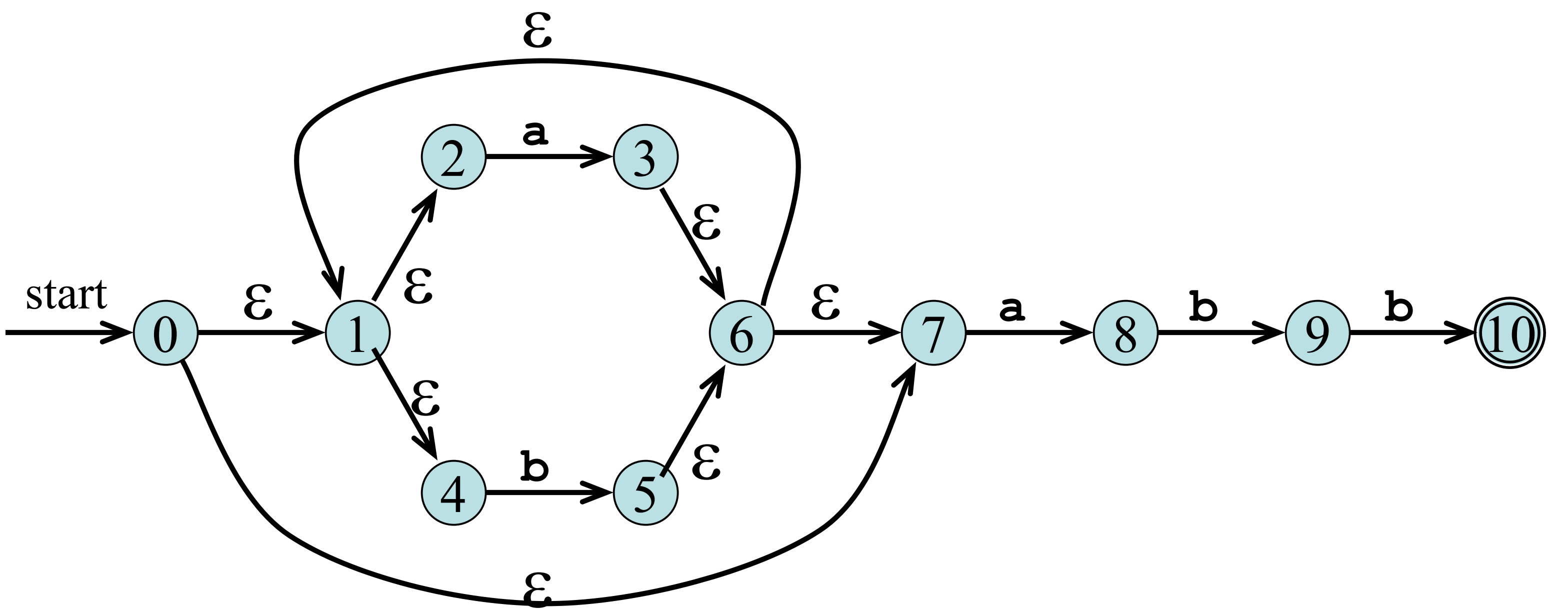
Simulating an NFA using ε-*closure* and *move*

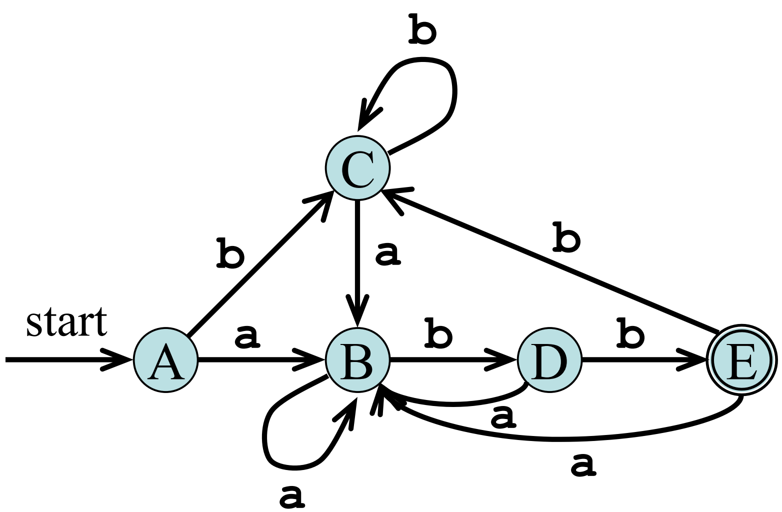
$S := \varepsilon\text{-}closure(\{s_0\})$
$S_{prev} := \emptyset$
$a := nextchar()$
while $S \neq \emptyset$ **do**
    $S_{prev} := S$
    $S := \varepsilon\text{-}closure(move(S,a))$
    $a := nextchar()$
end do
if $S_{prev} \cap F \neq \emptyset$ **then**
    execute *action in* S_{prev}
    return "yes"
else **return** "no"

The Subset Construction Algorithm

Initially, ε-*closure*(s_0) is the only state in *Dstates* and it is unmarked
while there is an unmarked state *T* in *Dstates* **do**
    mark *T*
    for each input symbol $a \in \Sigma$ **do**
        U := ε-*closure*(*move*(*T*,*a*))
        if *U* is not in *Dstates* **then**
            add *U* as an unmarked state to *Dstates*
        end if
        Dtran[*T*,*a*] := *U*
    end do
end do

Subset Construction Example 1





Dstates

A = {0,1,2,4,7}

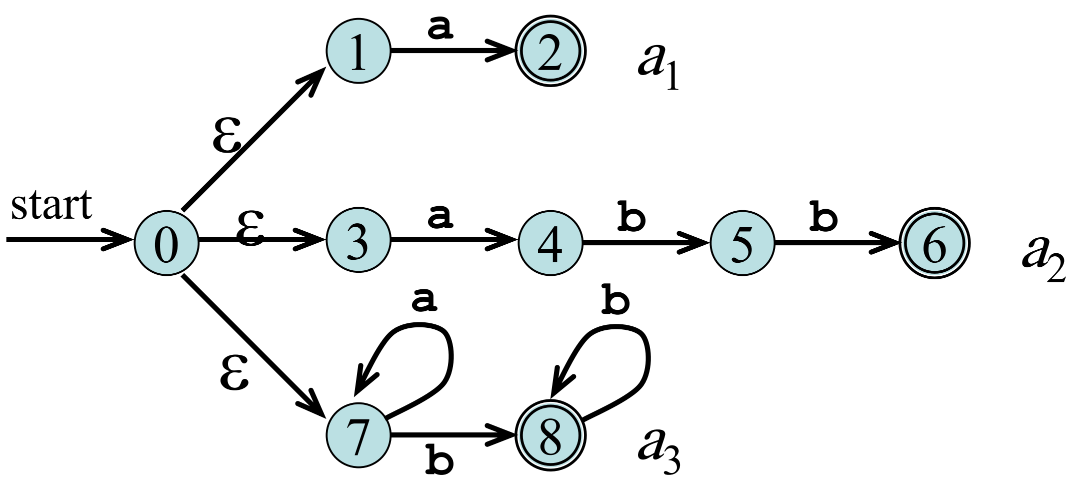
B = {1,2,3,4,6,7,8}

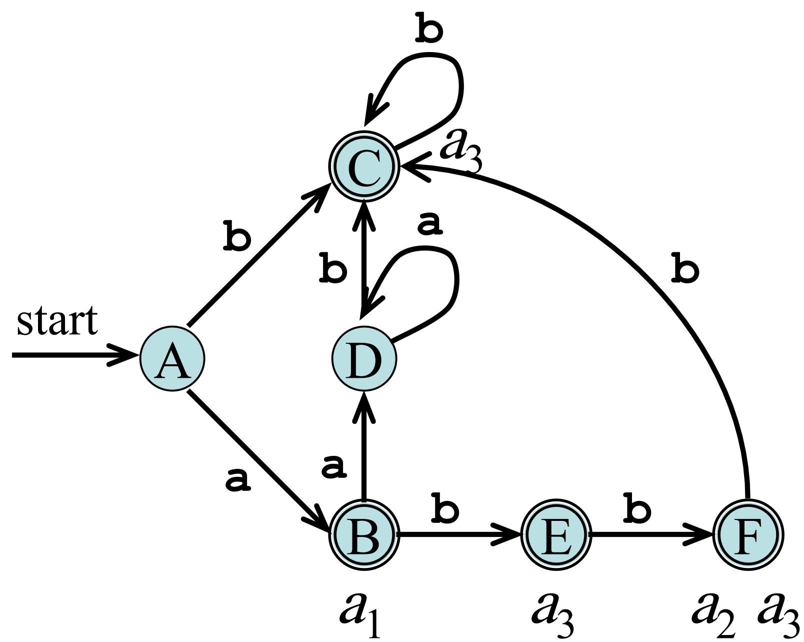
C = {1,2,4,5,6,7}

D = {1,2,4,5,6,7,9}

E = {1,2,4,5,6,7,10}

Subset Construction Example 2





Dstates

A = {0,1,3,7}

B = {2,4,7}

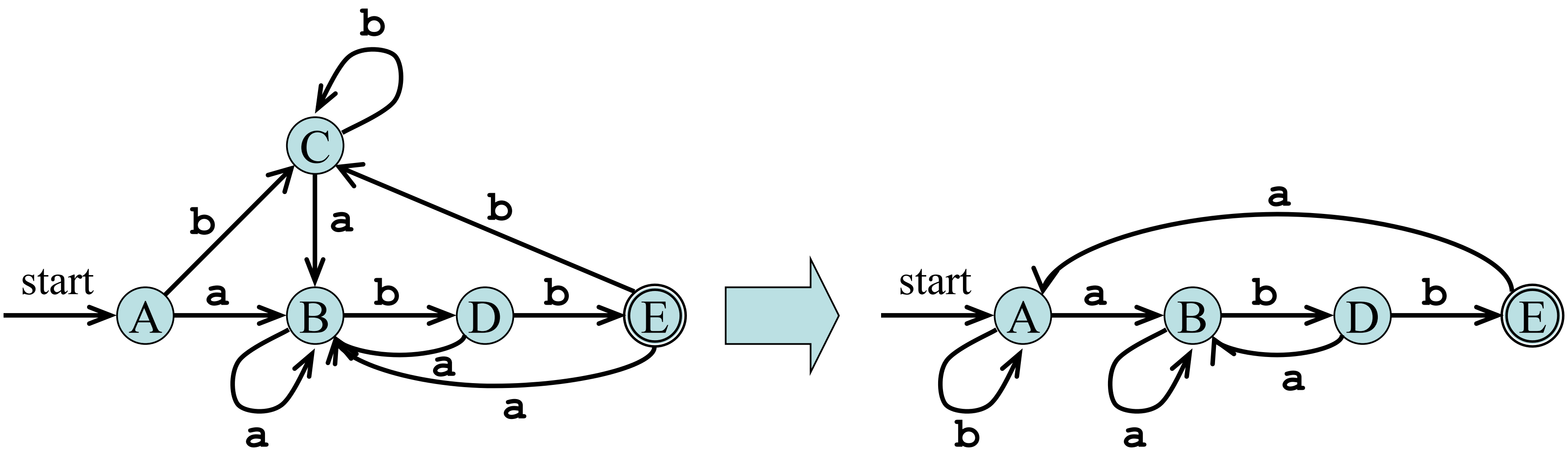
C = {8}

D = {7}

E = {5,8}

F = {6,8}

Minimizing the Number of States of a DFA



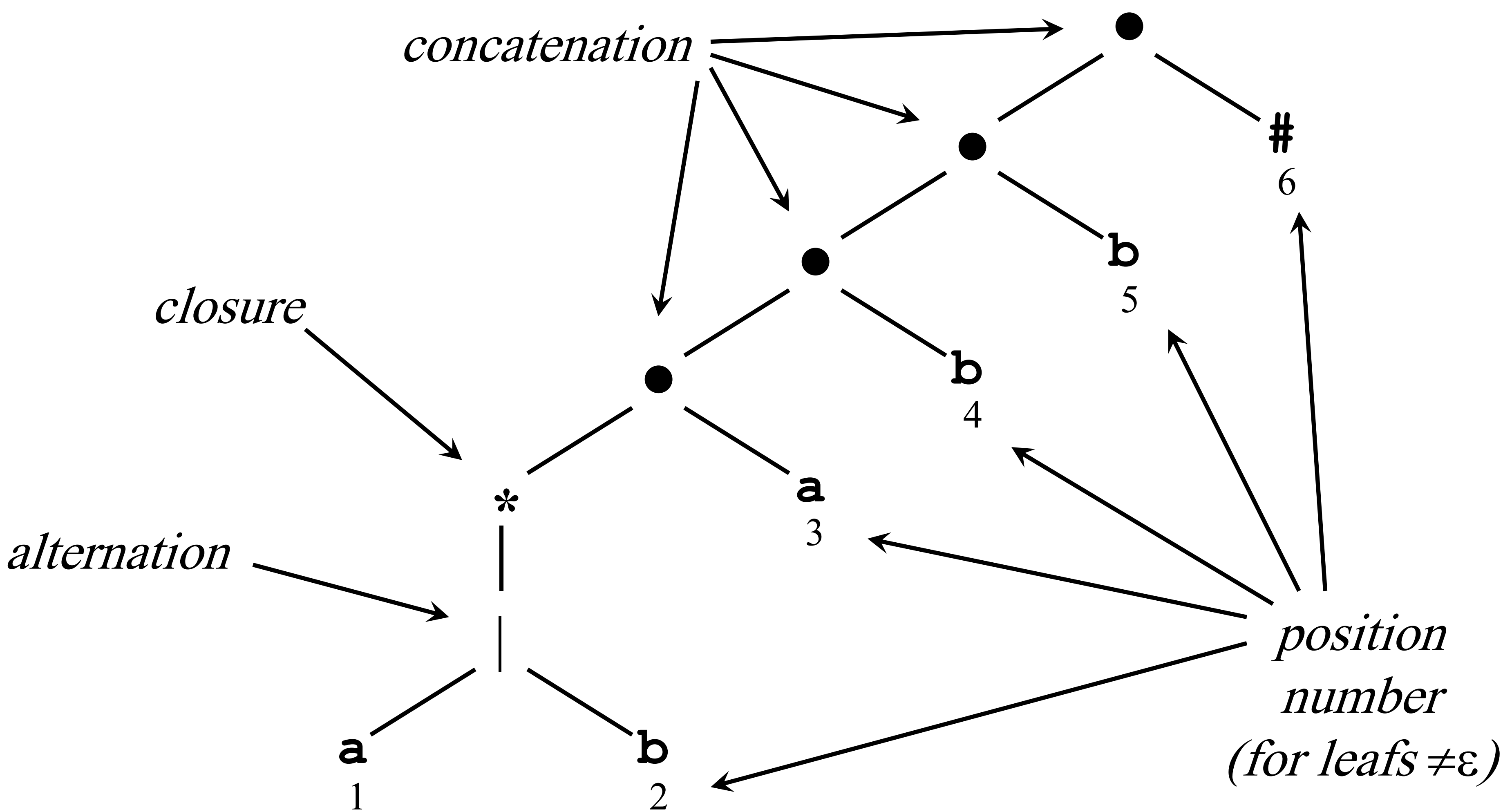
From Regular Expression to DFA Directly

- The *important states* of an NFA are those without an ε-transition, that is if $move(\{s\},a) \neq \emptyset$ for some a then s is an important state
- The subset construction algorithm uses only the important states when it determines $\varepsilon\text{-}closure(move(T,a))$

From Regular Expression to DFA Directly (Algorithm)

- Augment the regular expression *r* with a special end symbol # to make accepting states important: the new expression is *r*#
- Construct a syntax tree for *r*#
- Traverse the tree to construct functions *nullable*, *firstpos*, *lastpos*, and *followpos*

From Regular Expression to DFA Directly: Syntax Tree of (a|b)*abb#



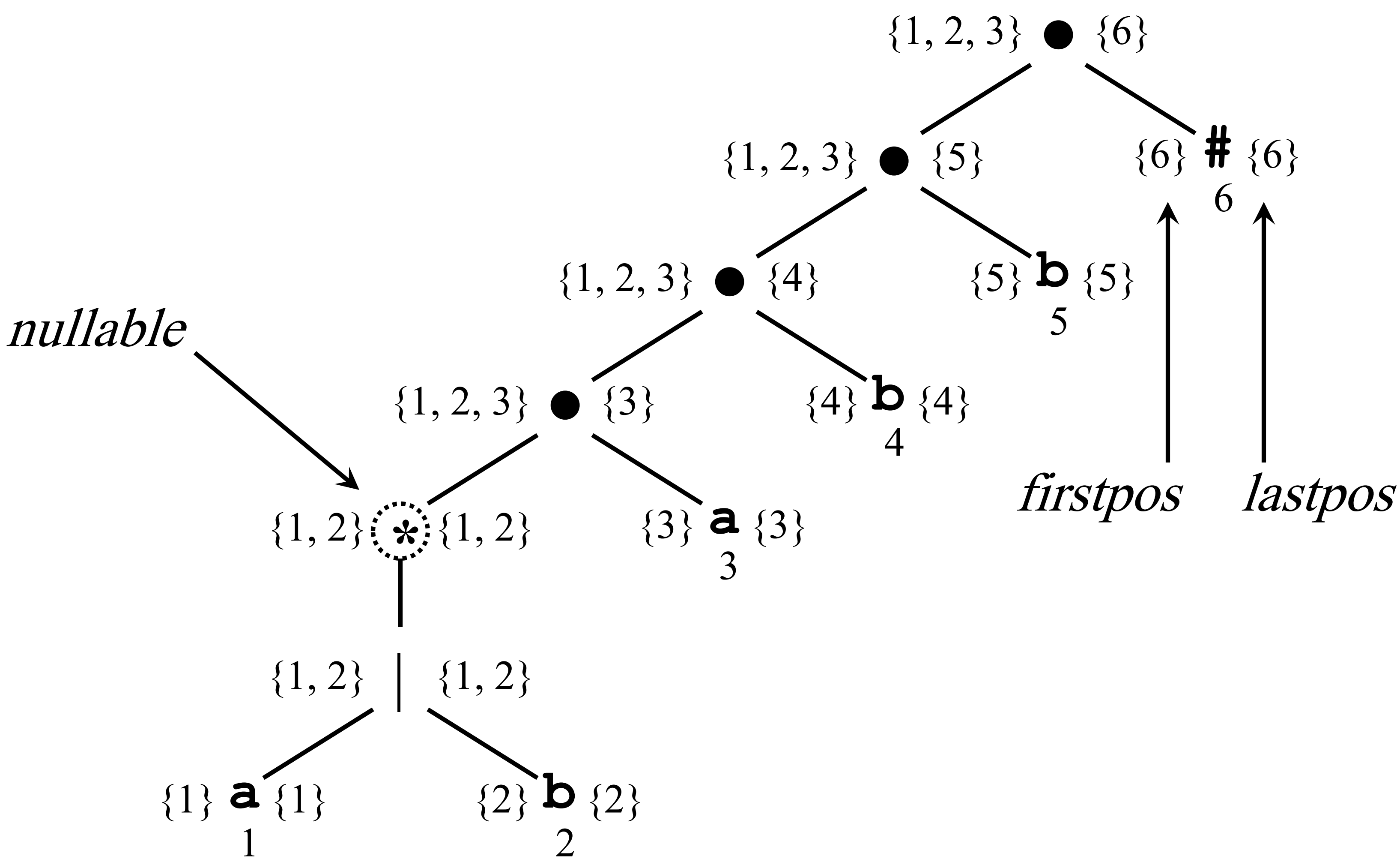
From Regular Expression to DFA Directly: Annotating the Tree

- *nullable*(*n*): the subtree at node *n* generates languages including the empty string
- *firstpos*(*n*): set of positions that can match the first symbol of a string generated by the subtree at node *n*
- *lastpos*(*n*): the set of positions that can match the last symbol of a string generated be the subtree at node *n*
- *followpos*(*i*): the set of positions that can follow position *i* in the tree

From Regular Expression to DFA Directly: Annotating the Tree

<table>
<tr><th>Node n</th><th>nullable(n)</th><th>firstpos(n)</th><th>lastpos(n)</th></tr>
<tr><td>Leaf ε</td><td>true</td><td>$\emptyset$</td><td>$\emptyset$</td></tr>
<tr><td>Leaf i</td><td>false</td><td>$\{i\}$</td><td>$\{i\}$</td></tr>
<tr><td>|
/ \
c_1 c_2</td><td>$\mathit{nullable}(c_1)$
or
$\mathit{nullable}(c_2)$</td><td>$\mathit{firstpos}(c_1)$
$\cup$
$\mathit{firstpos}(c_2)$</td><td>$\mathit{lastpos}(c_1)$
$\cup$
$\mathit{lastpos}(c_2)$</td></tr>
<tr><td>•
/ \
c_1 c_2</td><td>$\mathit{nullable}(c_1)$
and
$\mathit{nullable}(c_2)$</td><td>if $\mathit{nullable}(c_1)$ then
$\mathit{firstpos}(c_1) \cup$
$\mathit{firstpos}(c_2)$
else $\mathit{firstpos}(c_1)$</td><td>if $\mathit{nullable}(c_2)$ then
$\mathit{lastpos}(c_1) \cup$
$\mathit{lastpos}(c_2)$
else $\mathit{lastpos}(c_2)$</td></tr>
<tr><td>*
|
c_1</td><td>true</td><td>$\mathit{firstpos}(c_1)$</td><td>$\mathit{lastpos}(c_1)$</td></tr>
</table>

From Regular Expression to DFA Directly: Syntax Tree of (a|b)*abb#



From Regular Expression to DFA Directly: *followpos*

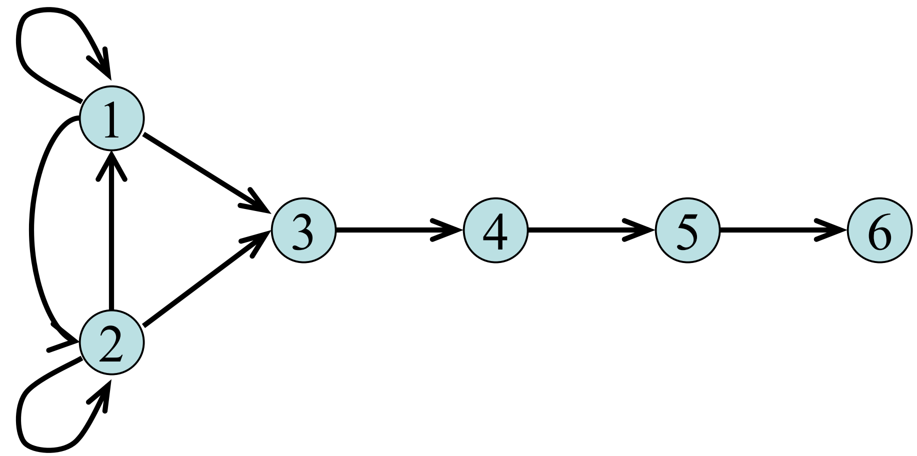
for each node n in the tree **do**
  if n is a cat-node with left child c_1 and right child c_2 **then**
    for each i in $lastpos(c_1)$ **do**
      $followpos(i) := followpos(i) \cup firstpos(c_2)$
    end do
  else if n is a star-node
    for each i in $lastpos(n)$ **do**
      $followpos(i) := followpos(i) \cup firstpos(n)$
    end do
  end if
end do

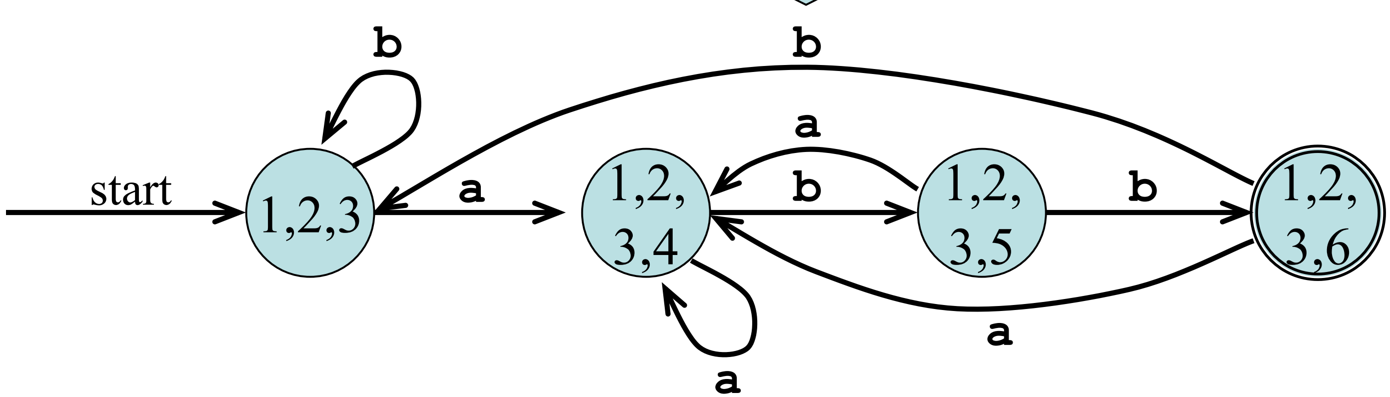
From Regular Expression to DFA Directly: Algorithm

s_0 := *firstpos*(*root*) where *root* is the root of the syntax tree
Dstates := { s_0 } and is unmarked
while there is an unmarked state *T* in *Dstates* **do**
        mark *T*
        for each input symbol $a \in \Sigma$ **do**
                let *U* be the set of positions that are in *followpos*(*p*)
                        for some position *p* in *T*,
                        such that the symbol at position *p* is *a*
                if *U* is not empty and not in *Dstates* **then**
                        add *U* as an unmarked state to *Dstates*
                end if
                Dtran[*T*,*a*] := *U*
        end do
end do

From Regular Expression to DFA Directly: Example

Node	*followpos*
1	{1, 2, 3}
2	{1, 2, 3}
3	{4}
4	{5}
5	{6}
6	-





Time-Space Tradeoffs

Automaton	*Space (worst case)*	*Time (worst case)*
NFA	$O(\lvert r \rvert)$	$O(\lvert r \rvert \times \lvert x \rvert)$
DFA	$O(2^{\lvert r \rvert})$	$O(\lvert x \rvert)$